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Question 1: This question contains **two** parts; Write only the final answers of them in the corresponding space provided.

(a) Find the values of x where the Dirac-delta function $\delta(\sin x)$ is non-vanishing. [1]

Answer: $x = n\pi$ with $n = 0, \pm 1, \pm 2, \pm 3, \ldots\ldots$

(b) Using the formula $\delta(y(x)) = \sum_i \frac{\delta(x - x_i)}{|\frac{dy}{dx}|_{x=x_i}}$, where x_i is the i^{th} root of $y(x) = 0$, evaluate

$$\int_{-\infty}^{+\infty} dx\ \delta(\sin x)\ e^{-|x|}\ .$$

[2]

Answer: $\frac{1 + e^{-\pi}}{1 - e^{-\pi}} = \coth(\pi/2)$.

Question 2: Consider an electric displacement vector $\vec{D} = zs\cos^2\phi\hat{z}$ units in cylindrical coordinates in some region of space. Determine the following:

(a) The free volume charge density at any point. [1]

Answer: $\rho_f = s\cos^2\phi$

(b) The total free volume charge enclosed within the cylinder of radius 1 unit with $-2 \leq z \leq 2$ units. [2]

Answer: $\frac{4\pi}{3}$ units.

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Question 3: This question consists of **three** parts. Write only the final answers at the respective spaces.

a) Consider a uniformly charged sphere of radius R with total charge Q. Find the electric field at a point $\vec{r}$ inside the sphere. [1]

Answer:

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^3} \vec{r}$$

b) Determine the z-component of the force on a volume element $d\tau$ at any position $\vec{r}$ in the northern hemisphere. [2]

Answer:

$$\left(\frac{3}{\epsilon_0}\right)\left(\frac{Q}{4\pi R^3}\right)^2 r\cos\theta \cdot d\tau$$

c) Using the above results find the net force that the southern hemisphere exerts on the northern hemisphere. [2]

Answer:

$$\frac{3Q^2}{64\pi\epsilon_0 R^2}$$

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Question 4: This question consists of **two** parts. Write only the final answers at the respective spaces.

a) Find the potential in the vicinity of an infinite straight line charge with uniform line charge density λ. [1]

Answer:

$$V = -\frac{\lambda}{2\pi\epsilon_0} \ln\left(s/s_0\right);$$ where s_0 is some length scale of the system and ref. pt. is chosen in such a way that $s/s_0 = 1$ at ref. pt.

b) The above infinite straight line charge is placed at a distance d above an infinite grounded conducting plane. Let's say that the wire runs parallel to x-axis, directly above the grounded conducting plane situated at $z = 0$.

(i) What is/are the boundary surface/s for the region $z > 0$? Also specify the boundary conditions necessary for solving the Poisson's equation. [0.5 + 0.5]

Answer:

* Boundary surfaces at $z = 0$ and $y^2 + z^2 >> d^2$
* Boundary conditions: $V = 0$ at $z = 0$
  $V \to 0$ as $y^2 + z^2 >> d^2$.

(ii) Find the potential in the region above the plane. [2]

Answer:

$$V = \frac{\lambda}{4\pi\epsilon_0} \ln\left[\frac{y^2 + (z+d)^2}{y^2 + (z-d)^2}\right]$$

(iii) Find the charge density σ induced at any point on the conducting plane. [2]

Answer:

$$\sigma = -\frac{\lambda d}{\pi(y^2 + z^2)}$$

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Question 5: This question consists of **three** parts. Write only the final answers at the respective spaces.

Consider an uncharged polarized dielectric sphere of radius R with "frozen in" polarization

$$\vec{P} = -\frac{\rho_0}{3}\left(1 - \frac{3r}{4R}\right)\vec{r}\,,$$

where ρ_0 is a positive constant. Find the following:

a) Surface and volume bound charge densities. [1 + 1]

Answer:
$$\sigma_b = -\frac{\rho_0 R}{12}$$
$$\rho_b = \rho_0\left(1 - \frac{r}{R}\right)$$

b) Magnitude of the electric fields as a function of distance from the center, both inside and outside the polarized sphere. ~~[1 + 2]~~ [2+1]

Answer:
$$\vec{E}_{in}(\vec{r}) = \frac{\rho}{3\epsilon_0}\left(1 - \frac{3r}{4R}\right)\vec{r}$$
$$\vec{E}_{out}(\vec{r}) = \vec{0}$$

c) The maximum value of the electric field $E_{\max}$ and the corresponding distance $r_{\max}$ from the center. [1 + 1]

Answer:
$$E_{max} = \frac{\rho_0 R}{9\epsilon_0}$$
$$r_{max.} = \frac{2R}{3}.$$

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Question 6: This question consists of **four** parts. Write only the final answers at the respective spaces.

Consider a vector field $\vec{F} = z\hat{i} + x\hat{j} + y\hat{k}$, and the part of the paraboloid, $z = \frac{x^2}{4} + \frac{y^2}{9}$, such that $z \leq 1$.

a) Find the unit vector on the curved surface of the paraboloid corresponding to traversing its contour at $z = 1$ in the positive (counter clockwise) direction. [1]

Answer:

$$\hat{n} = \frac{-\frac{x}{2}\hat{i} - \frac{2y}{9}\hat{j} + \hat{k}}{\sqrt{\frac{x^2}{4} + \frac{4y^2}{81} + 1}}$$

b) Determine the curl of the vector field $\vec{F}$. [0.5]

Answer:

$$\vec{\nabla} \times \vec{F} = \hat{i} + \hat{j} + \hat{k}$$

c) The positive flux of $\vec{F}$ through the above part of the paraboloid may be expressed as the double integral

$$\Phi = \iint_{\mathcal{R}} g(x, y)\, dR\,,$$

where $\mathcal{R}$ is the projection of the paraboloid on the xy-plane.

Determine (i) the equation of the region $\mathcal{R}$, (ii) the limits of the above flux integral, and (iii) the function $g(x, y)$. [1 + 1 + 1]

Answer:

(i) $\frac{x^2}{4} + \frac{y^2}{9} = \text{Const.} = 1$

(ii) $x \in [-2, 2]$, $y(x) \in \left[-3\sqrt{1-\frac{x^2}{4}}\,,\ 3\sqrt{1-\frac{x^2}{4}}\right]$

(iii) $g(x,y) = -\frac{x}{2} - \frac{2y}{9} + 1$

Given in the next page.

d) Determine the flux Φ. [1.5]

Answer:

$\cancel{6\pi}$

Given in the next page.

Q6 (c) $\Phi = \iint_S \vec{F}\cdot\hat{n}\, dS = \iint_R (\vec{F}\cdot\hat{n}) \frac{dx\,dy}{|\hat{n}\cdot\hat{k}|}$

$$= \iint_R g(x,y)\, dR$$

where R is the region on the xy-plane denoted by the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$, and the limits of the integration are

$$x \in [-2, 2],$$

$$y(x) \in \left[-3\sqrt{1-\frac{x^2}{4}},\ 3\sqrt{1-\frac{x^2}{4}}\right].$$

Finally, $g(x,y) = \frac{\vec{F}\cdot\hat{n}}{|\hat{n}\cdot\hat{k}|} = \frac{(z\hat{i}+x\hat{j}+y\hat{k})}{\cancel{\sqrt{\frac{x^2}{4}+\frac{4y^2}{81}+1}}}\cdot\left(-\frac{x}{2}\hat{i}-\frac{2y}{9}\hat{j}+\hat{k}\right) \times \cancel{\sqrt{\frac{x^2}{4}+\frac{4y^2}{81}+1}}$

$$= -\frac{zx}{2}-\frac{2xy}{9}+y$$

$$= -\left(\frac{x^2}{4}+\frac{y^2}{9}\right)\frac{x}{2}-\frac{2xy}{9}+y$$

or, $\boxed{g(x,y) = -\frac{x^3}{8}-\frac{xy^2}{18}-\frac{2xy}{9}+y}$

(d) $\Phi = -\int_{-2}^{+2} dx \int_{-3\sqrt{1-\frac{x^2}{4}}}^{3\sqrt{1-\frac{x^2}{4}}} dy \left(-\frac{x^3}{8}-\frac{xy^2}{18}-\frac{2xy}{9}+y\right) = 0$

since the above integral is odd.